

# An Improved Model for Noise Characterization of Microwave GaAs FET's

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**Abstract**—A simple, lumped-element noise model for microwave FET's is proposed where determination of the elements requires a minimal quantity of measured data. At the same time, the model is complete enough to yield good predictions of measured results. These predictions are in better agreement with measurements than those of a similar, recently published model.

## I. INTRODUCTION

MICROWAVE circuit designers in need of a noise model for GaAs FET's have a wide range of possibilities to choose from [1]–[7]. At one extreme are detailed analytical models and computer simulations of noisy devices. At the other are simple, empirical formulas fitted to measured results. This paper describes a modeling approach based on straightforward analysis and a modest quantity of measured results. The measured results used are broad-band microwave  $S$  parameters together with a single-frequency measurement of one noise parameter. As a result, the model is well suited for use in connection with microwave wafer probing of  $S$  and noise parameters.

Gupta *et al.* have also described a noise model whose purpose is to make efficient use of data from wafer probing [1], [2]. Their derivation of noise parameters from their device model is somewhat unconventional, and it appears to overlook an important aspect of FET behavior. The derivation presented in this paper follows a standard sequence, and it allows for all fundamental characteristics of FET operation. Comparison of results from the new model with results from measurements yields good agreement.

## II. THE FET NOISE MODEL

Fig. 1 is a schematic diagram of the microwave FET noise model. The model combines a lumped-element FET model with two uncorrelated noise sources. The first is the generator  $e_r$ , representing thermal noise in the gate series resistance. The second source is the generator  $i_c$ , representing random fluctuations in the drain current due to effects which are local to the drain–source channel. The lumped-element model contains enough detail to represent the essentials of FET small-signal behavior, including transconductance, drain–gate feedback, input

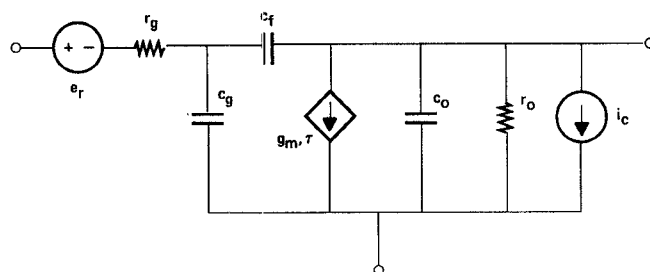


Fig. 1. Lumped-element noise model for microwave GaAs FET's.

loss, and finite output impedance. Analysis of the model is straightforward using standard methods [8], [9]. A portion of the analysis is presented, however, in support of the discussion of [1] and [2] contained in Section IV.

Many analytical approaches, e.g. Cappy [6], treat the power spectral density of  $i_c$  as constant over frequency. This makes it possible to define a frequency-independent channel noise conductance  $g_c$ , where

$$g_c = \frac{\overline{i_c^2}}{4kT}. \quad (1)$$

(The reference bandwidth in this equation, and in all similar equations of this paper, is 1 Hz.)

We first form the input and output short-circuit noise currents,  $i_1$  and  $i_2$ . The input short-circuit current is just due to thermal noise in  $r_g$ . The output short-circuit current is due to channel noise and to noise voltage induced across  $c_g$  by the input current. Hence, from inspection of Fig. 1,

$$\overline{i_1^2} = 4kTg_{in} \quad (2)$$

$$i_2 = \frac{y_{21}}{y_{11}} i_1 + i_c \quad (3)$$

where the  $y$  parameters  $y_{11}$  (with real part  $g_{in}$ ) and  $y_{21}$  may be readily determined from Fig. 1.

The next step is to reference the noise sources to partially correlated noise voltage and current generators at the input. The input noise current  $i_i$  and input noise voltage  $e_i$  are

$$i_i = \frac{-y_{11}}{y_{21}} i_c$$

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and

$$e_i = \frac{-i_1}{y_{11}} - \frac{i_c}{y_{21}}. \quad (4)$$

The correlation admittance  $Y_\gamma$  between  $e_i$  and  $i_i$  is

$$Y_\gamma = \frac{\overline{e_i^* i_i}}{e_i^2} = Ky_{11} \quad (5)$$

where

$$K = \frac{g_c}{|y_{21}|^2 r_g + g_c}. \quad (6)$$

A frequency-dependent conductance  $g_u$  can represent the uncorrelated part of  $i_i$ :

$$g_u \triangleq \frac{1}{4kT} \overline{|i_i - Y_\gamma e_i|^2} = K^2 g_{in} + (1 - K)^2 \left| \frac{y_{11}}{y_{21}} \right|^2 g_c. \quad (7)$$

The noise parameters now follow readily. The noise resistance, optimum source admittance and susceptance, and minimum noise factor are, respectively,

$$\begin{aligned} r_n &= r_g + \frac{g_c}{|y_{21}|^2} \\ g_0 &= \left( \frac{g_u}{r_n} + K^2 g_{in}^2 \right)^{1/2} \\ b_0 &= -K \operatorname{Im}(y_{11}) \\ F_0 &= 1 + 2r_n(Kg_{in} + g_0). \end{aligned} \quad (8)$$

### III. COMPARISON TO MEASURED RESULTS

The model in Fig. 1 contains a set of lumped elements and two noise generators. Applying the model to a real device requires making enough measurements to determine all the element and generator values. There are many different combinations of measurements which can provide this information, depending on the available equipment.

One combination suggests itself as being particularly convenient for use in connection with a microwave wafer probe. This consists of broad-band  $S$  parameters together with a single-frequency measurement of  $b_0$ . The  $S$  parameters give information to determine the lumped elements and the noise voltage generator. The single measurement of  $b_0$  gives additional information to determine  $g_c$ . Other combinations are possible, since any of the noise parameters could provide information to determine  $g_c$ . Use of  $b_0$  is convenient, however, because of its simple form in (8).

A test of the model has applied it to  $S$  and noise parameter data measured from a low-noise GaAs FET. The FET is a production device fabricated at the Watkins-Johnson Company. It has a uniformly doped active layer prepared by molecular beam epitaxy, and gate dimensions of  $0.3 \times 200 \mu\text{m}^2$ . Collection of test data took place at a drain-source bias of 4 V and 18 mA. Measured data include broad-band  $S$  parameters together with noise

TABLE I  
NOISE MODEL PARAMETERS FOR  $0.3 \times 200 \mu\text{m}^2$  GAAs FET

Element	Value
$r_g$	10.9 $\Omega$
$c_g$	0.14 pF
$c_f$	0.018 pF
$g_m$	30.0 mS
$\tau$	2.0 ps
$c_0$	0.044 pF
$r_0$	260 $\Omega$
$g_c$	17.8 mS

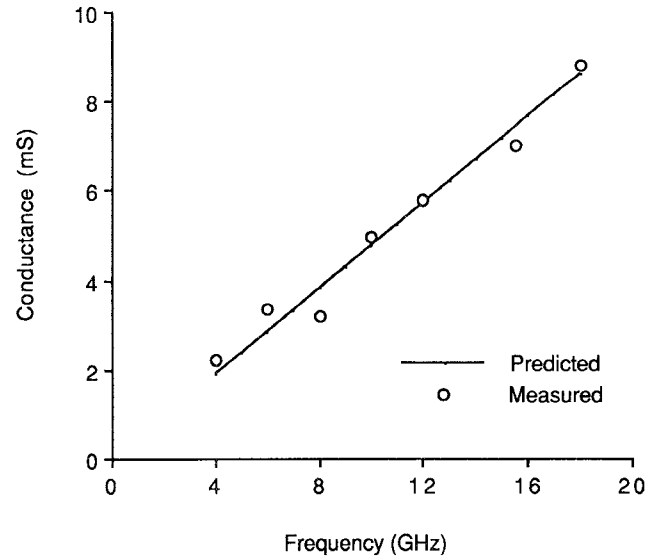


Fig. 2. Measured (O) and predicted (—) values of optimum source conductance versus frequency for the FET summarized in Table I.

parameters from several frequencies ranging from 4 to 18 GHz. The measurements took place with the FET mounted in a microstrip test fixture. Use of the de-embedding method of Peck [10] made it possible to reference the measured data to the bonding pads on the FET chip. Noise parameter measurements used a manual system similar to the one described by Pucel *et al.* [11]. Fitting the lumped-element model to measured  $S$  parameters was performed using Supercompact, while fitting noise parameters to the noise measurement results made use of the usual least-squares fitting technique [12].

Table I shows element values which resulted from applying the model of Fig. 1 to measured data. All items in the table except for  $g_c$  were determined from measured  $S$  parameters. Determination of  $g_c$  made use of the  $b_0$  value measured at 12 GHz. That measured value was  $-7.63$  mS, implying through (8) that the value of  $K$  was 0.65 at this frequency. This  $K$  was then used in (6) to determine the frequency-independent value of  $g_c$  shown in the table.

Figs. 2–5 compare measured values of  $g_0$ ,  $b_0$ ,  $F_0$ , and  $r_n$  to modeled values. The measured and modeled noise parameters agree well throughout the 4 to 18 GHz range, although the measured values of  $r_n$  show considerable variance. The single-frequency value of  $b_0$ , combined with broad-band  $S$  parameters, provided sufficient information

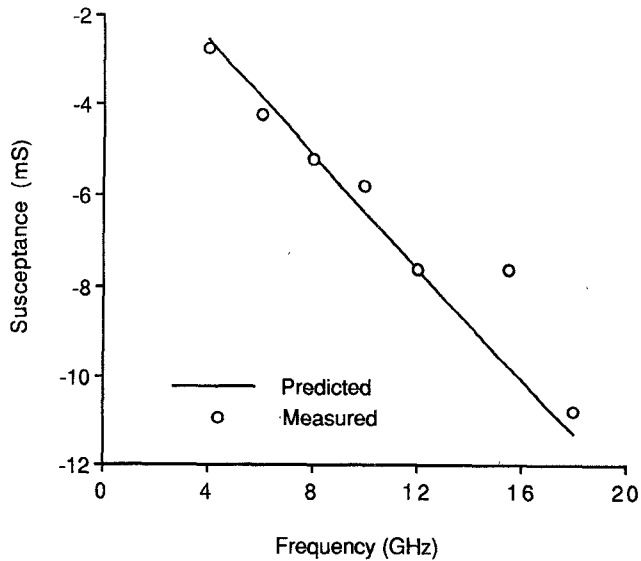


Fig. 3. Measured (○) and predicted (—) values of optimum source susceptance versus frequency for the FET summarized in Table I.

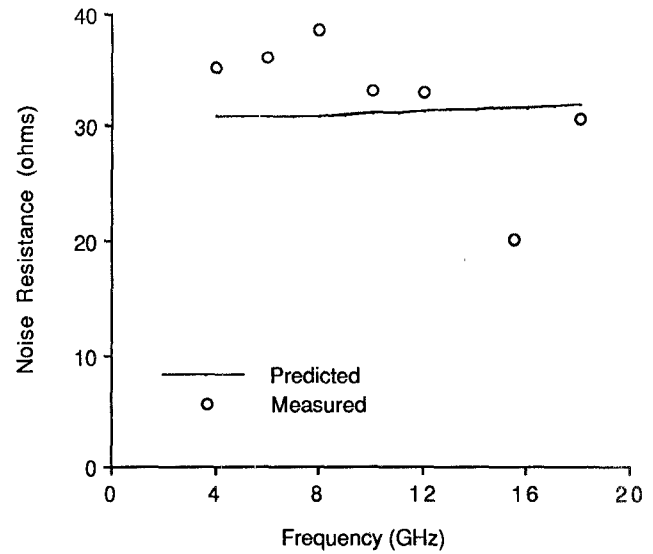


Fig. 5. Measured (○) and predicted (—) values of noise resistance versus frequency for the FET summarized in Table I.

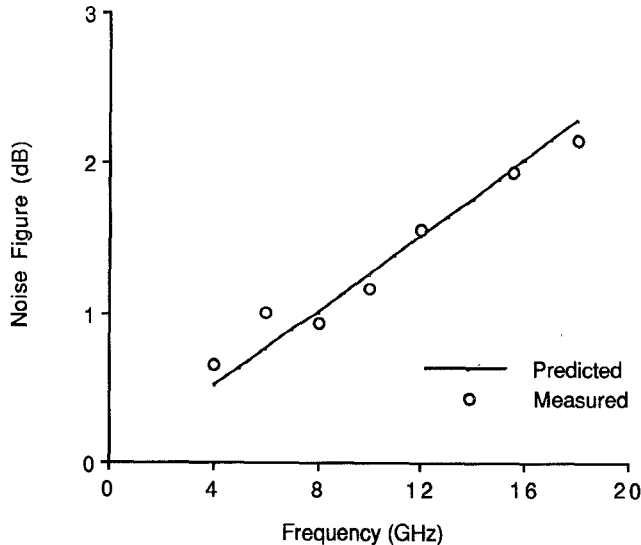


Fig. 4. Measured (○) and predicted (—) values of minimum noise figure versus frequency for the FET summarized in Table I.

to build a successful broad-band model. It should be noted, however, that the noise parameters depend strongly on  $r_g$ , while the  $r_g$  dependence of the  $S$  parameters is relatively weak [7]. Therefore it is somewhat fortuitous that fitting  $r_g$  to  $S$  parameters only led to such good results.

#### IV. RELATIONSHIPS TO OTHER LUMPED MODELS

Gupta *et al.* [1], [2] have also presented a lumped-element noise model for GaAs FET's. The form of the model is similar to Fig. 1, although  $c_f$  is omitted. The method used by the authors of [1] to analyze their model differs from the standard methodology used in this paper, and it leads to different definitions of the equivalent input noise and current sources. In spite of these differences, the general form of their result, as presented in

[1, eq. (6)], is equivalent to the results presented in this paper, provided  $c_f$  is assumed zero.

References [1] and [2], however, give no analysis of the correlation admittance defined there, relying instead on assumptions that any correlation between the input and output short-circuit currents is negligible. These assumptions are in contradiction to (3), which states that the total output short-circuit current includes a substantial term proportional to the input short-circuit current, resulting primarily from the transconductance. Ignoring this term and following the analysis of (4)–(7) leads to the conclusion that  $K$  is identically 1, and, among other consequences, that  $b_0$  is equal to the imaginary part of  $y_{11}$ . In the case of the measurement results presented in Section III, this prediction of  $b_0$  is in error by about 50% at every measurement frequency.

Pospieszalski [7] recently published an in-depth description of a lumped-element model based on measured data and of its relationship to analytical models. His model is equivalent to the one presented here, although it accounts for channel noise in a different manner, assigning an effective temperature to the small-signal conductance of the channel rather than defining a channel noise conductance at ambient temperature. Reference [7] shows good agreement between measured and modeled noise parameters of an HEMT at both room and cryogenic temperatures. The results indicate that this type of model is valid for HEMT's as well as for standard GaAs MESFET's and that the range of validity of the model in Fig. 1 can extend over temperature with appropriate adjustments to (1)–(8).

Pospieszalski also points out that the noise parameters depend strongly on the value of  $r_g$ , whereas the dependence of the  $S$  parameters is relatively weak. As a result, it is difficult, using  $S$  parameters alone, to determine  $r_g$  with sufficient accuracy for a noise model. He therefore uses noise parameter information to assign a noise tem-

perature to  $r_g$  which may be different from the physical temperature of the FET chip.

An alternative to Pospieszalski's procedure would be to use measured noise data to help determine the value of  $r_g$  while assuming its noise temperature to be equal to the physical temperature. In either approach, construction of a noise model would require the use of more measured noise data than the single-frequency value of  $b_0$  employed in Section III. In combination with measured  $S$  parameters, several data points relating noise factor and source admittance would be sufficient to determine  $r_g$  and  $g_c$ . These could be obtained from a fixed-tuned circuit by varying the measurement frequency, thus eliminating the need for a tuning element in experimental determination of noise parameters.

## V. SUMMARY AND CONCLUSIONS

A broad-band noise model for microwave FET's has been described. The model consists of small-signal lumped elements together with two noise sources. A measurement of broad-band  $S$  parameters plus a single-frequency measurement of optimum source susceptance can yield enough information to determine the model, although greater accuracy will be obtained from using additional noise data to determine the precise value of the gate resistance. The model's predictions match well with measured noise parameter data for a high-performance GaAs FET over a wide frequency range.

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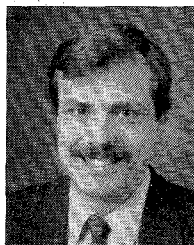
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